Homework is a sandwich. I give you the problem and the answer, so there's no credit for these. What's inside the sandwich is your job—make it hefty and tasty.
— Mario Warburton

In Mario Warburton's algebra classroom, homework is not about finding answers, which are all in the back of the book. It's about learning the mathematics to get those answers. It's also about recognizing when you really don't understand and need a little help.

At the start of the school year, students learn that what counts in algebra homework is quality of work, not quantity. They will face half as many problems as usual—ten instead of twenty to thirty. The problems will be a mix of skill, fluency, conceptual understanding, and applications problems; almost all will be odd-numbered—with answers in the back. But students will be expected to do every one of them correctly, checking their own work. If by class time they still cannot figure some out, they will get a "partial credit" stamp on their homework calendar.

No stigma is attached to these partial credit stamps, however. "Score and fix" and "Learn from your mistakes"—these are the mantras for the first class sessions each fall as students review their papers in class. All must do their initial work in pencil and use red pens for corrections (see fig. 1). They practice locating answers, using sticky notes to bookmark the text's answer section.

Once students transition to full self-checking, they must have their papers out at the start of class, all problems completed and marked with red stars or X's (fig. 2 shows a sample of student-corrected work). As the teacher goes around the room recording homework completion on monthly homework calendars, students self-assess, with some saying that they still do not quite understand the problems and do not deserve full credit.
If enough students seem to be having difficulties, Ms. Warburton will review strategies for solving the type of problem that is an issue, but students themselves must be responsible for their own solutions. They have two weeks or until the month’s end to earn a homework upgrade to full credit, getting help with troublesome problems in the after-school tutoring sessions, which are held twice weekly and are well attended.

Periodically, students self-evaluate the homework process using a simple rubric (see fig. 3). Most say that they like the system and that it works well for them. Each fall a few ask to be given credit for “all that work” that they put into incorrect solutions and complain about the “extra work” of doing them over correctly. Ms. Warburton’s response: “It’s not practice makes perfect—wrong practice is worse than none! Perfect practice is what really makes perfect.”

In a midyear homework reflection, some 64 percent of students reported following the self-check procedures. More than 80 percent said that having answers helped them learn. As one student explained, “If you’re doing it wrong and catch it, you don’t have to make a whole situation a do-over.” Another added, “If it’s right, it encourages you to keep going.” By the end of a year, complaints are rare, and the majority of students score 80 percent or higher on homework; in 2012 the median was 90 percent.

The Longfellow Middle School algebra program, codeveloped by Ms. Warburton and her colleague Juliana Jones, is rapidly becoming a legend in the district. Each year its students have outscored district and state averages by significant margins. U.S. Education Secretary Arne Duncan and Congresswoman Barbara Lee have praised the school’s efforts and in 2010 initiated the Longfellow program for outstanding achievement in mathematics and the impressive percentage of students of color performing at or above grade level.

Since then, things have only improved. Last year, while other schools in the district moved their lowest-performing eighth graders into algebra, Longfellow continued to place the full spectrum of students into algebra. Even with these struggling students, Longfellow outscored the other schools by a substantial margin, with 75 percent achieving “proficient” or “advanced” ratings (see fig. 4). Longfellow eighth graders consistently list algebra as among their “best-ever” classes and openly strive for these ratings this diverse Title I school with more than 50 percent of students receiving free or reduced-cost lunch.

Clearly, homework is a part of a bigger picture including excellent teaching, skillful classroom management, warm relationships with students, and a carefully nurtured social environment that makes mathematics achievement a positive student norm. But that is only the homework process is a critical piece of Ms. Warburton’s overall system, communicating a psychologically empowering level of trust and confidence in students’ ability to self-correct and continuously improve.

In my own role as a two-day-a-week volunteer tutor in the program, I see an impressive level of student-to-student help and support. Most remarkable is the quality of that interaction, which focuses on the process for arriving at answers. Students are not thinking about the Common Core Standards for Mathematical Practices, but they are certainly helping one another “make sense of problems and persevere in solving them” and learning to “understand the approaches of others in solving complex problems” (Standard 1, COSS 2010, p. 6). Similarly, by comparing their answers with those in the text, they are “learning to monitor and evaluate their progress and change course if necessary.”

Further enhancing the quality of the student exchanges is the teacher’s endorsement of multiple paths to a solution. There is only one right answer, but there are many right ways to attain it, so learning from a neighbor does not have to mean copying his or her exact procedure. In solving the simple equation 2(x + 2) = 16, for example, it is equally acceptable to divide both sides by 2 to distribute first. Equations, tables of values, and graphs are all acceptable routes to a solution in word problems involving initial fees (e.g., entry or membership) plus a per unit rate (of time, rides, etc.). During daily mathematics warm-ups, typically involving types of problems currently being studied, students first work alone, then compare approaches in five minutes of collaborative learning before starting in a whole-class discussion of the alternatives.

"History for mistakes?" Mistakes are considered a normal and valuable part of a learning process that has its inevitable bumps and setbacks. What is important is how students respond to frustrations and what they learn as they correct errors. This message is reinforced through the integration connection between homework and testing procedures. Quizzes in alternate weeks are composed entirely of problems from recent homework. Any major discrepancy between quiz scores and homework scores is an occasion for student self-reflection: "Am I learning the mathematics behind the homework or just going through the motions?"

Following every test is a class assignment to correct (in red pen) any solutions that received less than full credit. Students can raise a poor test grade up to a 70 (C - level) by acquiring help in math homework; others students to correct the missed problems. Once students

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Fig. 3 This rubric can serve as a template for students’ daily homework reflection.

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>10</td>
</tr>
<tr>
<td>2007</td>
<td>22</td>
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<tr>
<td>2008</td>
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<td>2009</td>
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<td>2010</td>
<td>64</td>
</tr>
<tr>
<td>2011</td>
<td>67</td>
</tr>
<tr>
<td>2012</td>
<td>75</td>
</tr>
</tbody>
</table>

Fig. 4 This chart shows the approximate percentage of Longfellow eighth graders scoring "proficient" or "advanced" on the state algebra test over a seven-year period.

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Reflecting on last night's algebra homework...

1. Did I check my answers in the back of the book?
2. Did I check each answer immediately after each problem?
3. Did I catch any mistakes?
4. Was I able to figure out what I did wrong?
5. Did I fix my work (not just the answer) in red pen?
6. Are there any problems I still don't understand?
7. What is my plan for learning those problems? (Tutoring, Family-Friend, No Plan, Learned Already)
8. Did I write my final score as a fraction at the top of the page?
9. Did I turn in my homework answers help me learn? Comments?

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Step 4: Repeat this process for each homework problem.

Step 5: At the top of your homework, write a fraction showing how many of the problems you understand (1st or 2nd try both count) out of how many problems were assigned.
Reinforcing the toughness ideology is a residual belief that mathematics ability is innate and in short supply: “You either have it, or you don’t.”

Algebra teachers have often seen themselves as gatekeepers, making sure that those who “don’t have it” are not passed along to certain failure in advanced mathematics classes. Assigning homework with answers in advance clutters head on with this belief system.

The consequences are substantial. Since the earliest days of Bob Moses’ Algebras Project (see Moses and Cobb 2001), algebra has been recognized as a key gateway to higher education and economic success, a crucial focus of efforts to narrow the racial and socioeconomic achievement gap. Algebra homework has the potential to widen that gap sharply when more advantaged students successfully complete it, whereas the less advantaged enter a downward spiral of unfinished work, failed tests, negative attitudes, and shrinking effort—what Goldberg (2007) calls the “homework trap.”

But unless in-school mathematics time is greatly expended at the expense of other subjects, the extra practice and self-assessment of homework remain critical parts of mastering algebra, particularly for students with relatively weak mathematics background.

So it is essential that algebra teachers design homework systems that encourage the kinds of behaviors and attitudes that lead to long-run success in mathematics. If they fail, the students for whom homework is most essential will be those least likely to complete it.

CHANGING THE HOMEWORK PARADIGM

At a time when the debate continues over whether homework is overused, optional, or essential or favors well-off students over those with little home support, we must understand ways in which effective homework strategies can help narrow the achievement gap. Vatterott (2009, p. 94) argues convincingly that “the old paradigm of difficult, grade-assigning homework creates practical and motivational obstacles that converge to form the perfect storm for student failure” for those who do not fit the traditional mold of well-prepared student and supportive family. To make a homework system productive for all students, it must encourage mathematical thinking and skill development; provide useful, nonjudgmental feedback; enhance student responsibility for learning; help students assess awareness of their strengths and weaknesses; and promote student self-confidence and motivation for learning.

The homework-as-sandbox system does all those things. In much of what educators must currently do in this standardized test-dominated era, answers have greater weight than the thinking behind them. Machine-scored multiple-choice tests treat a lucky guess the same as a laboriously worked-out answer. This is not the case in Ms. Warburton’s classes. With both problem and answer given, completing homework is a bit like solving a maze. Students start anywhere; they only finish anywhere to start and where to end, so they try promising mathematics paths until one works. No one nickers when a student takes a wrong turn or rereturns steps. Feedback is prompt and concerned. The fear is that tomorrow’s homework check will reveal an embarrassing mistake in a student’s thinking. Not being able to figure something out is an expected part of learning (see fig. 1); instead of hoping that no one will notice, students can take the initiative and ask for help.

Once students learn such effective problem-solving approach, they earn full credit for that learning. There are no “getchas” lurking in algebra class—it’s a friendly place.

CONCERNS, QUIBBLES, AND CAVEATS

What about all those concerns about rigor and high standards or possible cheating? Let’s start with cheating. Cheating is possible in any system in which students complete work beyond the direct supervision of a teacher, but it is most likely to occur when the stakes are high, workload is heavy, and students doubt their own ability to do the work correctly. In the homework-as-sandbox system, stakes are low because work can be redone for full credit and homework performance is reported in a work habits grade, separate from the algebra proficiency grade. The workload is reasonable, and students get prompt and frequent affirmation of their ability to solve problems independently.

Standards in the homework-as-sandbox system algebra classes are amazingly high. Most students achieve real proficiency, and even those who do not still make remarkable progress. They are never let off the book for learning, as is common in “rigorous” classes where failing a test or falling behind can lead students to withdraw effort completely. I have heard high school students say, “What’s the use? I’ll just take the class over again.” One of my former students, taking algebra for the fourth time, had so internalized his sense of impending failure that he could not believe that he might pass without cheating.

Students in Ms. Warburton’s classroom have not experienced failure, and she is determined to ensure that they will not. The incentives for attending tutoring sessions range from the warm fuchia tea and positive environment to exhilarations in the classroom and at recent conferences. Even students who know in late May that they will be retaking algebra in ninth grade show up for tutoring, work hard, and complete homework.

Is this a perfect system? Not quite, and in any case this is still evolving—this year’s class has more emphasis on mindset coaching plus consideration of increased weight for cumulative tests that show mastery of material missed on earlier tests. But in its present form, the homework-as-sandbox system surpasses the many alternatives I have seen in a long educational career.

Last spring, a newly minted mathematics Ph.D from nearby University of California at Berkeley was visiting Ms. Warburton’s after-school “tea and tutoring” session, and I asked her about her experience in college mathematics courses. “The class,” she said, “I learned the most were ones where I had answers to the problem sets,” was her response. After all, at the upper reaches of mathematical thinking, brilliant insights often come where the answers are known, but the steps for attaining them must be logically demonstrated. Why can’t this process begin more often in algebra class?

ACKNOWLEDGMENTS

I wish to thank Marlo Warburton and her Longfellow Middle School students, who model the vision of effective algebra homework presented in this article. Ms. Warburton, marlowarburton@berkeley.net, is a Math for America Master Teacher at the University of California at Berkeley.

REFERENCES